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TVICA - Time Varying Independent Component Analysis and Its Application to Financial Data

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TVICA – Time Varying Independent Component Analysis and Its Application to Financial Data*

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Abstract

Source extraction and dimensionality reduction are important in analyzing high dimensional and complex financial time series that are neither Gaussian distributed nor stationary. Independent component analysis (ICA) method can be used to factorize the data into a linear combination of independent components, so that the high dimensional problem is converted to a set of univariate ones. However conventional ICA methods implicitly assume stationarity or stochastic homogeneity of the analyzed time series, which leads to a low accuracy of estimation in case of a changing stochastic structure. A time varying ICA (TVICA) is proposed here. The key idea is to allow the ICA filter to change over time, and to estimate it in so-called local homogeneous intervals. The question of how to identify these intervals is solved by the LCP (local

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change point) method. Compared to a static ICA, the dynamic TVICA provides good performance both in simulation and real data analysis. The data example is concerned with independent signal processing and deals with a portfolio of highly traded stocks.

JEL code: C14; C58; G17

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1 Introduction

Source extraction and dimensionality reduction are among the primary goals of multivariate financial time series analysis, which helps to extract features and to find latent relations of risk drivers from high dimensional and complex portfolios. With increasing dimension and larger piles of data, attainment of these goals can be challenging.

Conventional statistical methods based on Gaussianity and stationarity do the job of simultaneous dimension reduction and stochastic factor identification. Principal component analysis and factor analysis are the tools here. The assumption of stationarity and Gaussianity is questionable though for the stochastic description of financial data. The Gaussian distribution cannot be used to mark tail dependence of risk factors and it fails in providing the empirical facts like heavy tailedness, volatility clustering and intertemporal dependence of cross moments of order higher than 2. The practical need to retrieve the main driving stochastic factors is accentuated though in risk management and many other fields of applications and must be dealt with even without distributional assumptions. Although an eigenvalue decomposition of returns' covariance yields only uncorrelated factors, see e.g. Jolliffe (2002), Härdle and Simar (2012), together with the Gaussian distributional assumption, the factors are independent. Hence well-developed univariate methods can be applied to each independent (but actually uncorrelated) factor, without considering the dependence among the components anymore. This is one of the primary reasons why Gaussianity has been widely adopted though deviating from the empirical facts.

A recently developed multivariate statistical method, Independent Component Analysis (ICA), is different from the conventional approaches. ICA extracts Independent Components (ICs) using a linear filter but does not project onto the eigenvectors of the covariance matrix as PCA does. Instead, the independent factors are estimated via an optimization problem, in which the statistical cross dependence between the extracted ICs is minimized. While PCA maximizes the variance of the projected data under orthogonality constraints, ICA directly attacks the independence of the projected factor components. For the Gaussian case they coincide of course. A rich set of ICA algorithms exists e.g. FastICA proposed by Hyvärinen and Oja (1997) and other methods in Hyvärinen, Karhunen and Oja (2001). The dimensionality reduction feature of ICA is that it actually converts a high dimensional problem to a set of univariate ones, and all components are (at least approximately) independent. This technique has been implemented in stock returns analysis by Back and Weigend (1998), in risk management by Chen, Härdle and Spokoiny (2010), in high frequency analysis by Kouontchou and Maillet (2007), and in an intertemporal GARCH context by Wu, Yu and Li (2006). All these works demonstrate a nice performance of the ICA method, with applications to financial data that are not Gaussian distributed.

One essential assumption though is common to these papers: the observed series as well as the ICs are stationary and the linear filter is the same for the entire time series. As a consequence, the dynamics of cross dependence is constant over time which in light of the ever occurring turbulences in financial markets is questionable. In order to demonstrate how the performance of ICA is affected, if the stationarity assumption is violated, consider three ICs, each normal-inverse Gaussian (NIG) distributed (a heavy-tailed distribution, see Barndorff-Nielsen (1997) for more details). The NIG distributional parameters are actually calibrated from the empirical distribution of three ICs estimated for the log returns of Home Depot (HD), Hewlett-Packard (HPQ) and IBM. We present more returns later in our real data section. Two ICA filters, A_1 and A_2 are used for generating a realistic example series, corresponding to two different time periods: 3rd September 2008 to 31th August 2009 (a period with market turbulence), and 30th July 2004 to 29th December 2006 (a relatively quiet period):

$$A_1 = 10^{-3} \begin{pmatrix} 0.6 & 13.0 & 6.2 \\ 3.8 & 2.7 & 13.0 \\ 7.9 & 5.9 & 4.8 \end{pmatrix}, \quad \text{and} \quad A_2 = 10^{-3} \begin{pmatrix} -0.1 & 0.8 & 5.3 \\ 7.0 & 1.9 & 1.6 \\ 0.1 & 4.2 & 1.1 \end{pmatrix}.$$

The three NIG distributed ICs, $\mathbf{Z}_t \in \mathbb{R}^3$ produce the observed series $\mathbf{X}_t = A_t\mathbf{Z}_t$, in which $A_t = A_1$ for the first 300 observations and $A_t = A_2$ for the rest. Figure 1 displays the theoretical values of the ICs and the errors of the estimated ICs using either a Time Varying ICA method or a static ICA method. The static ICA assumes that the filter is constant over the whole time period, while the Time Varying ICA (TVICA) reacts to the change point of the filter and respectively estimates ICs based on separated locally homogenous samples. Apparently, the TVICA method benefits from adapting to local inhomogeneity and its error process therefore has a much narrower spread than that of the static ICA. The RMSE (root mean square errors) of the estimated ICs also indicates a good performance of the TVICA in terms of accuracy, with values of 0.886 (static) and 0.201 (time varying) respectively. When considering only the time period after change, the difference becomes even larger,

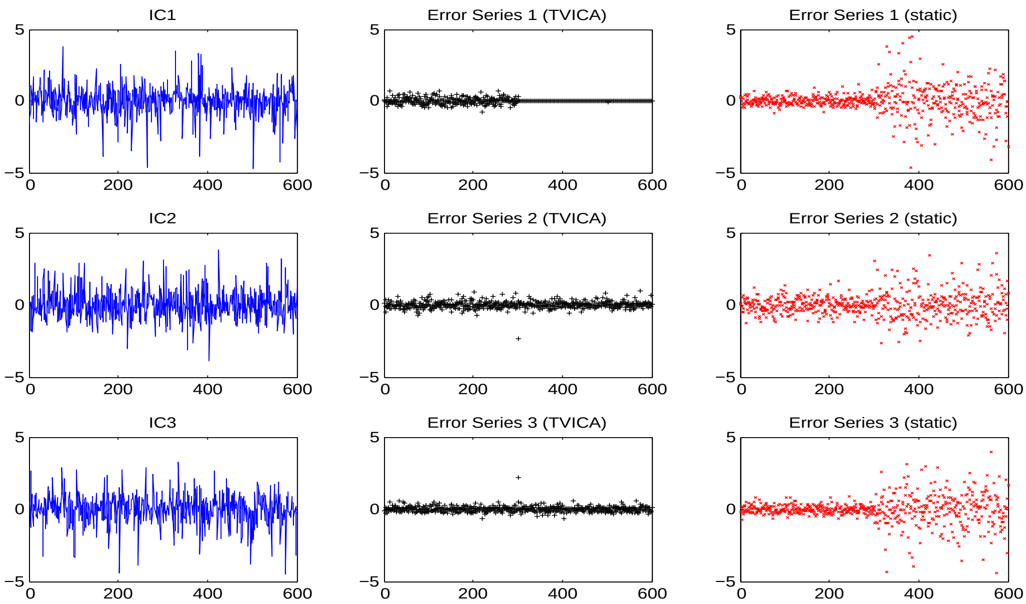

Figure 1: Demonstration based on three simulated series $\mathbf{X}_t = A_t \mathrm{IC}_t$, where $A_t = A_1$ for $t = 1, \ldots, 300$ and changes to A_2 after then. From left to right it displays the theoretical values of the ICs, the errors of the recovered ICs that are estimated over the separated locally homogeneous samples (TVICA) or over the whole sample (static ICA). The TVICA has a much narrower error spread than the static ICA.

with values of 1.196 (static) and 0.160 (time varying).

The above (reality driven) example makes it clear that one not only needs a non-Gaussian low dimensional factor extraction but also a technique that locally (in time) identifies a "trust interval" over which one can safely do ICA. The importance of identifying such an interval of approximate stationarity is often under-evaluated. Improving the quality of IC extraction for varying intervals, when dynamics changes over time, is the aim here. The little demonstration above indicates that the TVICA method is preferred in the case. The question is of course how to identify the locally homogeneous intervals in practice! Matteson and Tsay (2009) gave an answer by allowing the mixing matrix to vary over time via a smooth function of some transition variables. This idea is similar to time-varying models proposed in the volatility and co-volatility literature, see e.g. Baillie and Morana (2009), Scharth and Medeiros (2009). Also it resembles time variation models incorporating changes via Markov-Switching or mixture multiplicative error specifications that have been proposed by e.g. Hamilton and Susmel (1994), So, Lam and Li (1998), Lanne (2006). These techniques though take a globally given mechanism for this time variation, in contrast to e.g. Mercurio and Spokoiny (2004) using a local change point approach. The approach is different from the existing ones in the sense that it is data-driven and applicable for various kinds of breaks (macroeconomic or political changes) with different magnitudes and abrupt or smooth types. Neither prior information (on say states of the market) nor distributional assumption is required. It motivates us to develop a local estimation approach for ICA.

Here a time varying ICA (TVICA) framework is put into action, where the mixing matrix (linear filter) is allowed to change over time without imposement of a global structure. For each time point we determine a "trust interval" by conducting a sequence of tests on a structural change. The selection is controlled by a set of critical values. In this selected trust interval one performs ICA. The TVICA method is completely a data driven approach for filter and homogeneity determination.

The remainder of the paper is structured as follows. The next section presents in detail the time varying (constrained) ICA approach and the estimation procedure. Section 3 investigates the performance of the proposed approach along with a simulation study, and the real data analysis is reported in Section 4. Section 5 summarizes our findings and discusses an outlook to future work.

2 How TVICA works

Suppose that there are p assets with log returns $\mathbf{X}_t = \{x_1(t), \dots, x_p(t)\}^\top$. The aim is to factorize the financial returns into a linear combination of independent components $\mathbf{Z}_t = \{z_1(t), \dots, z_p(t)\}^\top$. The TVICA approach is based on:

$$\mathbf{X}_t = A_t \mathbf{Z}_t \tag{1}$$

where A_t is a $p \times p$ (time varying) matrix. In the static ICA approach, $\mathbf{X}_t$ in (1) is assumed to be stationary and $A_t = A = \text{const.}$ i.e. to be time homogeneous. In the TVICA approach, the linear filter A_t is time dependent and the estimation of ICs is customized under *Local Homogeneity* for any time point of interest.

Local homogeneity means that, for any particular time point t there exists a past time interval $I_t = [t - m_t, t]$, over which the linear filter A_t is approximately constant, i.e. $A_s \approx A$, $\forall\ s \in I_t$. Given t and its past information, the challenge is of course to determine I_t (or m_t) – the "trust interval of local homogeneity". In order to rise to this challenge, the Local Change Point (LCP) detection approach of Mercurio and Spokoiny (2004) is applied. Note that the LCP approach is data-driven and nests the above mentioned "smooth transition" and "regime switching" techniques used in earlier literature. Based on the identified trust interval, TVICA can provide more accurate performance than using a constant ICA filter.

2.1 The LCP method

In this section, we present the LCP detection procedure to identify the interval of local homogeneity at time point t. The estimation of the TVICA is carried out via the (quasi) maximum likelihood method by treating the linear filter or its inverse as unknown parameter. Suppose for a moment that an interval of a constant filter (*homogeneity*) $I_t = [t - m_t, t]$ is given at time point t, where m_t indicates the length of the interval. Then with pdf $f_j(z_j)$ of IC z_j, $j = 1, \ldots, p$, the pdf of $\mathbf{X}$, according to Jacobian transformation, is:

$$f_{\mathbf{X}}(x_1, \ldots, x_p | B_t) = \left\{ \prod_{i=1}^{p} f_j(z_j) \right\} \times |\det\, B_t|,$$

where B_t is the inverse of A_t. With $B_t = (b_{1t}, \ldots, b_{pt})^\top$, this gives:

$$f_{\mathbf{X}}(x_1, \ldots, x_p | B_t) = \prod_{j=1}^{p} f_j(b_{jt}^\top \mathbf{X}) \times |\det\, B_t|. \quad (2)$$

The log-likelihood function on the interval I_t is:

$$L(I_t, B_t) = \sum_{s=t-m_t}^{t} \sum_{j=1}^{p} \log\{f_j(b_{jt}^\top \mathbf{X}_s)\} + (m_t + 1) \log |\det\, B_t|, \quad (3)$$

and the MLE is denoted as $\widetilde{B}_t$.

Relaxing this situation of a constant (static) filter to *local homogeneity* on I_t means that B_t (or A_t) does not deviate too much from being constant in I_t. The deviation of this constant parameter is measured by a small modeling bias (SMB). The SMB quantifies the divergence of a time varying model relative to a static model, for details see Spokoiny (2011), Härdle, Panov, Spokoiny and Wang (2012). Take now a family of nested intervals, $I_0 \subset I_1 \subset \cdots \subset I_{K-1} \subset I_K$ (the subscript t is omitted here for simplification of notation), the longer the length of intervals, the smaller the variance of the estimator but the higher the bias. The LCP approach selects the longest interval of local homogeneity that has the smallest variance (a trust interval) given an SMB bound.

The identification of the trust interval at time t is done via a sequential algorithm. At the first step, the interval I_0 is accepted as a trust interval. Next for an interval I_k, $k = 1, \ldots, K$, the procedure is to sequentially screen a subinterval $J_k = I_k \setminus I_{k-1} = [t - m_k, t - m_{k-1})$ and check for a possible change point – an indication of non-homogeneity – in the interval with "new" information. The interval I_k is accepted if every point in J_k is tested to be insignificant as a location of change point. One continues this way until a change point is detected or the longest interval I_K is reached.

More specifically in the k-th step, given J_k as the testing interval we choose $I = [t', t'']$ to be a superset of J_k that includes some neighboring observations of J_k. Then for each point $t \in J_k$, we split the interval I into two sub-intervals, with $I' = [t', t)$ and $I'' = [t, t'']$. Note that $I = I' \bigcup I''$ and $I' \bigcap I'' = \emptyset$. Figure 2 demonstrates the relation of the intervals used in the testing procedure. Let $L_I(B)$ denote the log-likelihood

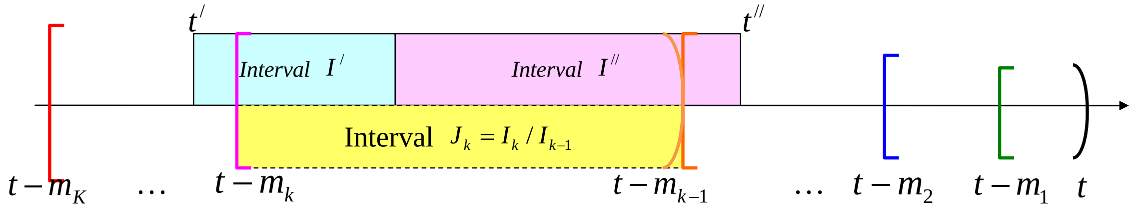

Figure 2: Local change point detection procedure.

function for the observations in I. The LCP method employs a likelihood ratio test at all $t \in J_k$ to examine "a possible change point" over the whole interval J_k:

$$T_{I,t} = \max_{B'',B'} \{L_{I''}(B'') + L_{I'}(B')\} - \max_{B} L_I(B). \tag{4}$$

The maximum (over t) of (4) is the proper statistic:

$$T_k = \max_{t \in J_k} T_{I,t} \tag{5}$$

If T_k is greater than a critical value η_k, the null hypothesis of local homogeneity on I_k is rejected. The critical values $\{\eta_k\}$ for $k = 1, \ldots, K$ are computed via Monte Carlo simulation, since the distributional properties of (5) are (even asymptotically) unknown. The details are described in Section 2.3.

The formal definition of the LCP algorithm is as follows:

1. Initialization: The null shouldn't be rejected on I_0. Denote the initial homogeneous estimate by $\widehat{B}_t^{(0)} = \widetilde{B}_t^{(0)}$.

2. Set $k = 1$. While $T_k \leq \eta_k$ and $k \leq K$,
   update the present homogeneous estimate by $\widehat{B}_t^{(k)} = \widetilde{B}_t^{(k)}$ and set $k = k + 1$.

3. Final Estimate: $\widehat{B}_t = \widehat{B}_t^k$, which is actually the MLE over the longest interval of local homogeneity.

It is worth mentioning that the numerical complexity of the LCP algorithm is not high. In the computation for a simulation data including 10 time series, with 610 sample points for each and a family of 6 nested intervals (Section 3), it takes about 10 minutes on a PC with 2.67GHz Intel(R) Core(TM) i7 CPU.

2.2 Finding ICs in a selected interval

Given an identified interval of local homogeneity, i.e. $B_t = B$, (quasi) maximum likelihood estimation (MLE) is used to obtain ICs. For the MLE approach, the Kullback-Leibler (KL) divergence measures the difference between two joint density functions of $\mathbf{X}$, $f_{\mathbf{X}}(x_1, \ldots, x_p | B_t)$ under independence assumption, see (2), and $p_{\mathbf{X}}$

given the observations:

$$\begin{aligned} KL\{p_{\mathbf{X}} \| f_{\mathbf{X}}(x_1,\dots,x_p|B_t)\} &= \int p_{\mathbf{X}} \log \frac{p_{\mathbf{X}}}{f_{\mathbf{X}}(x_1,\dots,x_p|B_t)}\, d\mathbf{X} \\ &= H_{\mathbf{X}} - \int p_{\mathbf{X}} \log f_{\mathbf{X}}(x_1,\dots,x_p|B_t)\, d\mathbf{X}, \end{aligned}$$

where $H_{\mathbf{X}} = \int p_{\mathbf{X}} \log p_{\mathbf{X}} d\mathbf{X}$. The ICs are obtained via minimizing the KL divergence with respect to B_t. Note that $H(\mathbf{X})$ doesn't depend on B_t. Hence minimizing the KL divergence is equivalent to maximizing $\int p_{\mathbf{X}} \log f_{\mathbf{X}}(x_1,\dots,x_p|B_t)\, d\mathbf{X}$, where the latter is proportional to the log-likelihood function in (3).

Given that financial time series have heavy-tailed marginal distributions, one considers:

$$\log f_j(z_j) = \alpha - 2\log\cosh(z_j) = \alpha - 2\log\left[\frac{1}{2}\{\exp(z_j)+\exp(-z_j)\}\right], \qquad (6)$$

where α is a normalizing constant. Take the logarithmic derivative $g_j(z_j) = \frac{\partial}{\partial z_j}\log f_j(z_j)$:

$$g_j(z_j) = -2\tanh(z_j) = -\frac{2\{\exp(2z_j)-1\}}{\exp(2z_j)+1}, \quad \forall\, j = 1,\dots,p, \qquad (7)$$

For the Gaussian case $g_j(x) = -x\varphi(x)/\varphi(x) = -x$, is a linear function. From (7) one see that g_j flattens out and thus models heavy tails. Hyvärinen and Oja (1999) claim that a small misspecification of the density (6) doesn't affect the consistency of the ML estimator, and therefore we adopt the particular selection.

It is worth mentioning that a pre-whitening process is normally conducted before implementing ICA. The variances of ICs are not identifiable according to the definition of ICA. Without loss of generality, the variances of ICs are set to be one by whitening the observations. It is easy to show that the linear filter of the pre-whitened series will be an orthonormal matrix. This feature will help to select parameters of the TVICA method.

2.3 Selection of Hyperparameters

The TVICA method is driven by a small set of "adjustable screws" or hyperparameters that we present here.

Set of intervals: The family of intervals $\{I_k\}_{k=0}^{K}$ is either given or selected as:

$$I_k = [t - m_k, t],$$

where $m_k = m_0 a^k$ with a pre-specified initial length m_0 and a multiplier $a > 1$. The coefficient a controls the increasing length of the trust intervals. The starting value m_0 should be sufficiently small to ensure very brief local homogeneous intervals. A practicable choice of a and m_0 is discussed later in the simulation section of this paper. We may however already state here that the proposed algorithm is only weakly sensitive to the choice of this interval sequence.

Critical values: The critical values $\{\eta_k\}$ are calculated under the null, i.e. a homogeneous constant filter B^*. They are calculated from a stability condition, to be described below. This involves an additional parameter ρ that controls the error of first kind, i.e. a false alarm see (8) below.

Under the null (constant filter), one generates independent series and mixes them with a constant filter matrix A^* or its inverse $B^* = A^{*-1}$. The MLE of filter over the shortest interval I_0 of the sample is used here, as the interval is a priori assumed to be time homogeneous. It is interesting to mention that ICA conducts a prewhitening process to avoid non-uniqueness of ICs, which makes the filter's scaling free but doesn't change the underlying pattern of homogeneity or non-homogeneity. Therefore, the selection of the constant filter is not crucial for detecting change point. Later we will demonstrate that the method is stable with respect to the selection of A^*. Under the produced globally homogeneous situation, every interval is locally homogeneous and the longest interval I_K is the optimal choice. The aim here is to compute the critical values so that the adaptive estimate $\widehat{B}_t$ that is driven by the computed critical values doesn't deviate much from the constant filter, or simply, the SMB condition is satisfied.

For any $r > 0$, the fitted log likelihood with $B_t = B^*$ can be used to measure the divergence of the MLE $\widetilde{B}_t^{(k)}$ for $t \in I_k$:

$$\mathsf{E}_{B^*}|L(I_k, \widetilde{B}_t^{(k)}, B^*)|^r = \mathsf{E}_{B^*}|L(I_k, \widetilde{B}_t^{(k)}) - L(I_k, B^*)|^r,$$

and the largest divergence among all the intervals is denoted as:

$$R_r(B^*) = \max_{k \le K} \mathsf{E}_{B^*}|L(I_k, \widetilde{B}_t^{(k)}, B^*)|^r.$$

The parameter r specifies the loss function under the null. A choice of $r = 0.5$ for example corresponds to the ℓ_1 loss and it provides a stable and robust performance in the monte carlo simulation. For a given value r, $R_r(B^*)$ can be computed straightforwardly.

The constant is in practice unknown. We here mimic the situation by replacing it with the adaptive estimate $\widehat{B}_t$. Under the null of local homogeneity, the modeling bias is required to be bounded:

$$\mathsf{E}_{B^*}|L(I_K, \widetilde{B}_t^{(K)}, \widehat{B}_t)|^r \le \rho R_r(B^*), \quad t \in I_K, \tag{8}$$

where $\widehat{B}_t$ depends on the critical values $\eta_1, \ldots, \eta_K$. The parameter $\rho > 0$ is the test level parameter. Its selection reflects the expectation and preference of users. A small value of ρ indicates that one expects a small divergence of the estimate to a constant filter (the null), which leads to relatively large critical values and a rather conservative procedure for possible time variation. Increasing ρ would result in a decrease of the critical values and an increase of the sensitivity of the method to the changes of filter in the underlying process. It might therefore be interpreted as a "false alarm" indicator.

To calculate the critical values, we do the calibration in a sequential way. Notice that the sequential homogeneity tests accumulate uncertainty in estimation due to the increase in the degrees of freedom and therefore the probability of an interval being homogeneous decreases. To take this into account, it is suggested to adjust the hyperparameter ρ for an individual level of test at each step $k = 1, \ldots, K$, by e.g. assigning increasing weights k/K to increase the sensitivity to changes. In particular, the kth-step adaptive estimate $\widehat{B}_t^{(k)}$ on the interval I_k satisfies:

$$\mathsf{E}_{B^*} |L(I_k, \widetilde{B}_t^{(k)}, \widehat{B}_t^{(k)})|^r \leq \frac{k}{K} \rho R_r(B^*), \quad t \in I_k. \tag{9}$$

where $\widehat{B}_k^{(k)}$ depends on the critical values $\eta_1, \ldots, \eta_k$.

How does it work in detail? At the initial step $k = 0$, we set $\eta_0 = \infty$ by accepting the shortest interval. To specify the next critical value η_1, we set the values of $\eta_2, \ldots, \eta_K$ to be infinity. Then η_1 is selected as the minimum value to satisfy (9):

$$\mathsf{E}_{B^*} \left| L(I_k, \left\{ \widetilde{B}_t^{(k)}, \widehat{B}_t^{(k)}(\eta_1, \eta_2) \right\} \right|^r \leq \frac{1}{K} \rho R_r(B^*), \quad t \in I_k, \quad k = 1, \ldots, K.$$

We then continue to select η_k given $\eta_1, \ldots, \eta_{k-1}$ and set $\eta_{k+1} = \cdots = \eta_K = \infty$,

$k = 2, \ldots, K$. The value of η_k is determined such that:

$$\mathsf{E}_{B^*} \left| L(I_\ell, \left\{ \widetilde{B}_t^{(\ell)}, \widehat{B}_t^{(\ell)}(\eta_1, \ldots, \eta_k) \right\} \right|^r \leq \frac{k\rho R_r(B^*)}{K}, \quad t \in I_\ell, \quad \ell = k, \ldots, K.$$

It is worth mentioning here that the LCP procedure is robust w.r.t. variation of these hyperparameters. In the simulation study of Section 3, we give evidence for this claim.

3 Simulation

This section investigates the performance of the TVICA method in various scenarios. In particular, we assess its detection power under homogeneity and non-homogeneity (nonstationarity) with different kinds of change point. As long as the underlying processes are stationary without change point, LCP should select the longest interval in the estimation of ICs. If at least one change point exists, LCP must detect the change point. It is worth emphasizing that the proposed method is not to identify the exact location of all possible change points. Instead, it is to select, for any particular time point, the longest interval before the occurrence of the most recent change point. The ICs are then safely estimated over the identified interval of local homogeneity. Since the LCP procedure relies on hyperparameters (r, ρ), that have to be pre-determined, we also analyze the impact of the hyperparameters with various values. It turns out that they have little influence on the performance of TVICA.

The setup of the simulation scenarios is practical. We use 10 components of the DJ30 index to generate the simulation processes. The 10 stocks are The Home Depot (HD), Hewlett-Packard (HPQ), IBM, Intel (INTC), Johnson & Johnson (JNJ), JPMorgan Chase (JPM), Kraft Foods (KFT), Coca-Cola (KO), McDonald's (MCD) and 3M (MMM). The data spans from 14th January 2010 to 28th October 2010, over which ICA is conducted for the daily log returns as described in Sec 2.2. The obtained ICs are taken to be NIG distributed since this type of distribution works well in financial applications. Accordingly, 10 independent NIG distributed series are generated, with 610 sample points for each. The simulation processes are obtained by mixing these independent sources with time dependent linear filters A_t in different scenarios. There are 1000 repetitions for each scenario.

- Scenario HOMO is a homogeneous case, where the linear filter is set to be an identity matrix for all the points. In this simplest case, where the simulated series are independent and homogeneous, the longest interval is the optimal selection.
- Scenario JPLF includes a sudden change, with A_t jumping from $\breve{A}$ to an identity matrix at $t = 251$. The matrix $\breve{A}$ is estimated based on the above mentioned real data with 10 stocks.
- Scenario JPEM also includes a sudden change at $t = 251$. Instead of a change of the whole filter matrix, only one element of the filter changes. More specifically, the (2,1)–element of the linear filter changes from 3 to 0 such that the linear filter becomes an identity matrix.

- Scenario SLEM refers to the type of smooth changes. As an illustration, the $(2, 1)$-component of the linear filter is defined as:

$$A_t^{(2,1)} = \begin{cases} 3, & t \leq 220; \\ 3(1 - (t - 220))/160, & 220 < t < 380; \\ 0, & t \geq 380. \end{cases} \tag{10}$$

The set of the time intervals is defined with $m_0 = 200$, $a = 1.25$ and $K = 5$:

$$I_0 = 200,\ I_1 = 250,\ I_2 = 313,\ I_3 = 391,\ I_4 = 488,\ I_5 = 610,$$

which corresponds to investment horizons from one year to 2.5 years. The parameters r and ρ in the LCP procedure are assigned to be either $0.1, 0.5$ or 1. Totally there are 9 combinations of (r, ρ). The computation of critical values are based on generated homogeneous series, with 610 sample points for each. We repeat the generation 5000 times. Moreover, when screening a possible change point in the interval $J_k = I_k/I_{k-1}$, a superset is set to be an interval that also includes the neighboring 25 observations of J_k. The critical values for different sets of hyperparameters are displayed in Figure 3. For any set of (r, ρ), The critical values are decreasing w.r.t. interval length, corresponding to the fact that for long intervals, the null of local homogeneity tends to be rejected. Moreover, the set of critical values shifts downside for a large value of ρ, which reflects an expectation of non-homogeneity and the method is hence sensitive to change point.

The TVICA is conducted for different sets of (r, ρ) in different scenarios. The detection power is measured by the ratio of rejecting the null of local homogeneity over 1000 replications. We summarize how much and where the null is rejected. In particular, a value of 0 means the null is not rejected at all, which indicates a perfect local homogeneity. On the contrary, a ratio of 100% means the null is completely rejected. In the latter case, we are curious where the most recent change point is and respectively report the ratio at each interval. The results are reported in Table 1. Under the scenario of homogeneity (Scenario HOMO), the ratios are lower than 10%, if $\rho < 1$. For $\rho = 1$, the error of first kind is up to 26.8%, underlining our earlier comment on the role of ρ. This false alarm will encourage a selection of a relatively short interval, in which the variance of estimators may be large but the modeling bias

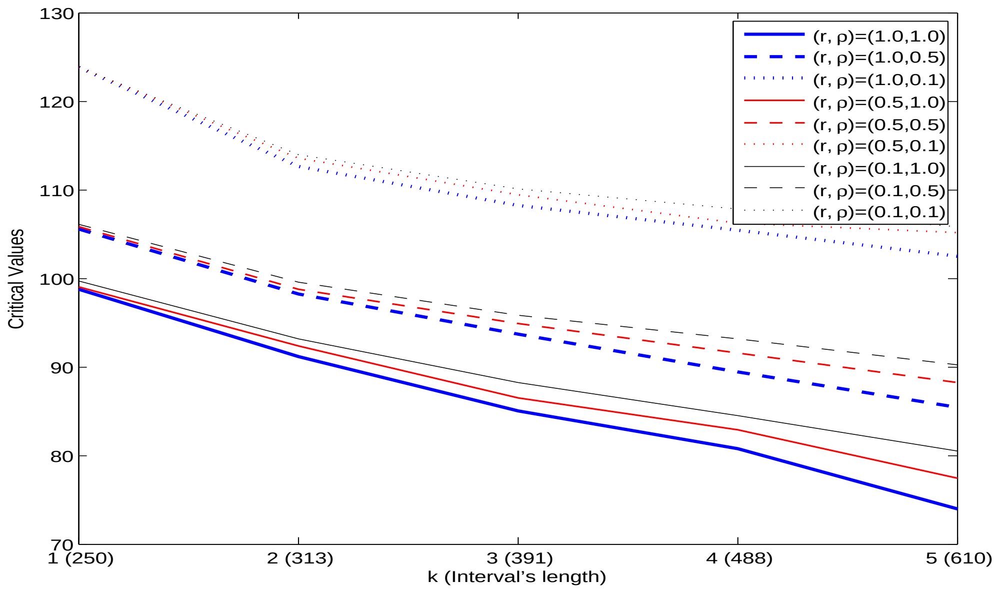

Figure 3: Critical values with parameters $r = 1, 0.5, 0.1$ and $\rho = 1, 0.5, 0.1$. The set of intervals is defined with $m_0 = 200$, $a = 1.25$ and $K = 5$. The length of intervals is listed in the parentheses. The computations are based on the generated independent series, with 610 sample points for each series and with 5000 replications.

is still small – the main concern of our study. Therefore the impact of false alarm is not serious. Under a scenario with change point(s), the ratios are 100% for all sets of (r, ρ). The location of the change point is nicely detected. For Scenario JPLF and JPEM with just one change point at $t = 251$, that is $t \in J_3 = I_3/I_2 = [219, 298)$, the ratios in I_3 are above 99%. For Scenario SLEM with the (2,1)–element of the filter matrix slowly changing over J_3, $J_2 = I_2/I_1$ and part of I_1, see (10), the total ratios in I_3 and I_2 is above 99%. For variation of ρ, the ratios have been reallocated between the intervals, however it doesn't affect much the selection of intervals. It just reflect the users' expectation and preference on homogeneity vs non-homogeneity. In general, the TVICA method works well and can select the interval of local homogeneity reasonably. Moreover, the performance of the TVICA method is very stable across r, given ρ is fixed.

		$r = 0.1$				$r = 0.5$				$r = 1.0$			
ρ	@	I_1	I_2	I_3	I_4	I_1	I_2	I_3	I_4	I_1	I_2	I_3	I_4
	HOMO		— 0.6 —				— 0.6 —				— 0.7 —		
0.1	JPLF	–	–	100	–	–	–	100	–	–	–	100	–
	JPEM	–	–	99.2	0.8	–	–	99.4	0.6	–	–	99.4	0.6
	SLEM	–	5.9	93.1	1.0	–	6.8	92.4	0.8	–	7.9	91.3	0.8
	HOMO		— 4.9 —				— 5.9 —				— 8.3 —		
0.5	JPLF	0.1	–	99.9	–	0.1	0.1	99.8	–	0.1	0.1	99.8	–
	JPEM	–	0.1	99.5	0.4	–	0.2	99.5	0.3	–	0.2	99.6	0.2
	SLEM	0.2	32.4	67.4	–	0.2	34.4	65.4	–	0.2	36.1	63.7	–
	HOMO		— 15.3 —				— 20.3 —				— 26.8 —		
1.0	JPLF	0.2	0.4	99.4	–	0.2	0.4	99.4	–	0.2	0.7	99.1	–
	JPEM	–	0.4	99.5	0.1	–	0.6	99.4	–	–	0.8	99.2	–
	SLEM	0.2	49.5	50.3	–	0.2	52.6	47.2	–	0.4	56.4	43.2	–

Table 1: The ratio of rejection (in percentage) of the LCP detection tests over 1000 replications. The definition of the scenarios is given in the text. Scenario HOMO is a homogeneous case, while the other scenarios include change points occurring at either the 3rd interval for Scenario JPLF and JPEM, or over part of I_1, $J_2 = I_2/I_1$ and $J_3 = I_3/I_2$. The results show the TVICA method works well with a strong detection power.

4 Real Data Analysis

In this section, we implement TVICA to the log returns of a portfolio with 10 stocks traded at NYSE: HD, HPQ, IBM, INTC, JNJ, JPM, KFT, KO, MCD, and MMM. Recall that the objective of our study is to safely retrieve independent signals out of complex time series that are neither Gaussian distributed nor stationary. We here attack the following questions. Does the proposed method detect a reasonable interval of local homogeneity given a particular time point? Are the signals that are estimated over the identified interval (approximately) independent? We consider here two time points, 1st August 2007 and 28th October 2010, which are respectively before and after the recent global financial crisis from 2008 to 2010. Using the same set of intervals in the simulation study, that is, $m_0 = 200$, $a = 1.25$ and $K = 5$, the task is to detect the most recent change point, if exists, over two periods [1st March 2005, 1st August 2007] and [30th May 2008, 28th October 2010], with 610 observations for each.

The first period corresponds to a relatively stationary (stochastically homogenous) period, during which no influential economic or financial events are observed. To justify, we assign a set of equal weights to the 10 stocks and recursively compute realized volatility for the point 1st August 2007:

$$\widetilde{\sigma}_t = \sqrt{\frac{1}{m}\sum_{t=T-m+1}^{T} r_t^2}, \quad 200 \leq m \leq 610,$$

where r_t denotes the return of the equally weighted portfolio. We start with 200 historical observations – the shortest interval, and continuously including one more, up to all the 610 observations – the longest interval in the computation. Figure 4 displays the realized volatility (dashed line) w.r.t. m, the length of the sample. The values are quite stable around 0.030, which indicates a stationarity over the whole period. Hence, the longest interval could be a reasonable selection.

The second period [30th May 2008, 28th October 2010] instead involves the stock market crash in 2008. Again, we recursively compute the realized volatility of the equally weighted portfolio over the period, see Figure 4. Apparently, the volatility process is not constant. It shifts from about 0.045, around the interval I_3, or when more than 391 historical observations are considered. It is interesting to see whether the TVICA method can detect the possible change and identify a reasonable interval of local homogeneity.

In our study, the hyperparameters (r, ρ) are set to be $(0.5, 0.5)$ and $(0.1, 0.1)$ as both provided good power for the scenario of homogeneity in the simulation study. In the computation of critical values, the hypothetically homogeneous B^* is set to be

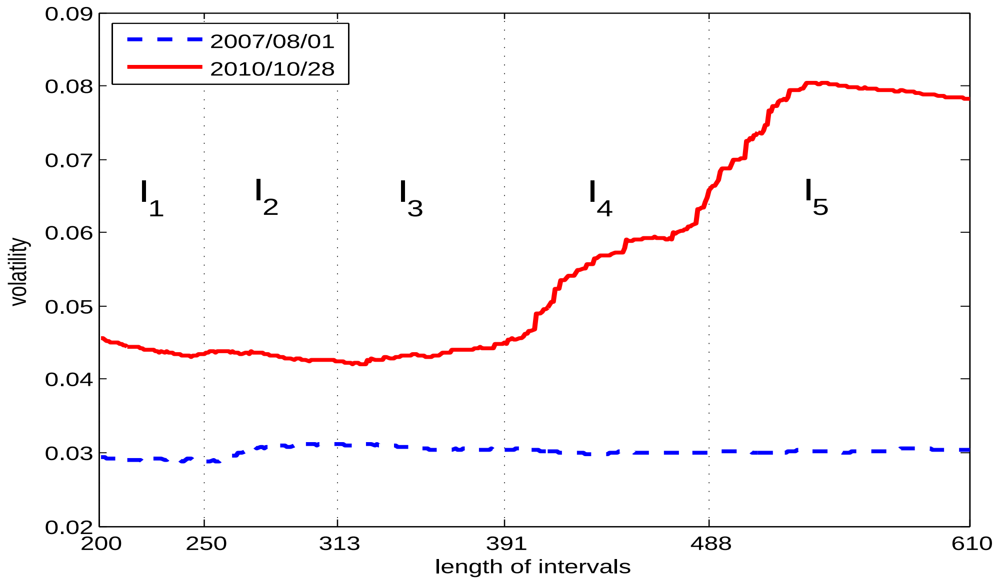

Figure 4: Realized volatility recursively computed for two days 1st August 2007 and 28th October 2010. The set of intervals with $m_0 = 200$, $a = 1.25$ and $K = 5$ is marked in the plot to highlight the underlying pattern across the intervals.

either the MLE in the shortest interval or an identity matrix. Table 2 reports the critical values as well as test statistics for every interval. For the 1st August 2007, the test statistics are insignificant so the null of local homogeneity is not rejected over the whole time period of [1st March 2005, 1st August 2007]. In other words, one can safely implement ICA in the time period. On the contrary, for the 28th October 2010, the null is rejected at the interval I_3, and the interval $[2009/08/05, 2010/10/28]$ is identified to be homogenous. The result is consistent to the above volatility analysis, in which a structure change initiates around the interval I_3.

For the two cases, the sets of critical values are close for different B^*, which justifies the selection of B^* is not crucial for detecting change point. Moreover, the critical values vary a little across the hyperparameters. However, the testing results are identical. It supports our claim that the proposed method is stable to variation of the hyperparameters.

	2005/03/01-2007/08/01					2008/05/30-2010/10/28				
	CV				T_I	CV				T_I
(r,ρ)	$(0.5, 0.5)$		$(0.1, 0.1)$			$(0.5, 0.5)$		$(0.1, 0.1)$		
B^*	MLE	Identity	MLE	Identity		MLE	Identity	MLE	Identity	
I_1	107.23	102.84	122.37	120.89	74.36	108.87	105.85	126.51	123.74	69.81
I_2	98.40	98.45	117.43	113.21	76.62	101.71	98.67	116.86	113.95	81.97
I_3	93.15	92.35	112.30	108.44	66.86	96.32	94.92	113.91	110.05	**265.35**
I_4	89.64	88.81	109.53	105.57	77.52	92.59	91.57	111.18	107.80	**469.99**
I_5	86.28	85.74	106.82	103.01	72.79	88.72	88.21	108.99	105.85	**205.60**

Table 2: The critical values and the test statistic for two experiments. The set of intervals for testing is defined as $m_0 = 200$, $a = 1.25$ and $K = 5$. The CVs are computed with respect to B^* equals the MLE in the shortest interval or an identity matrix. The hyperparameters are set to be $(r, \rho) = (0.5, 0.5)$ and $(r, \rho) = (0.1, 0.1)$. The critical value computations are based on the generate 10 independent series, with 610 sample points for each series and with 5000 replications.

Moreover, we use higher-order (4th order) cross-cumulants as a measure of statistical independence:

$$\mathrm{cum}(z_i, z_j, z_k, z_l) = \mathsf{E}(z_i z_j z_k z_l) - \mathsf{E}(z_i z_j)\,\mathsf{E}(z_k z_l) - \mathsf{E}(z_i z_k)\,\mathsf{E}(z_j z_l) - \mathsf{E}(z_i z_l)\,\mathsf{E}(z_j z_k),$$

where $z_.$ denotes the obtained (independent) signal process. If the signals are independent, the cross-cumulants are zero when i, j, k, l are not equal simultaneously. As a comparison, we also implement a static ICA over the longest interval (610 observations) and a dynamic PCA over the interval of local homogeneity that is identified in the TVICA method for the two points. The cross-cumulants are computed. Figure 5 displays the boxplots of all the cross-cumulants of the signals by using the TVICA, the static ICA and the dynamic PCA. For both the stationary and nonstationary cases, all the cross-cumulants balance around zero, with means closing to 0. But the dynamic PCs and the static ICs have wider spreads and more outliers, which attributes to either the gaussianity assumption or the stationarity assumption.

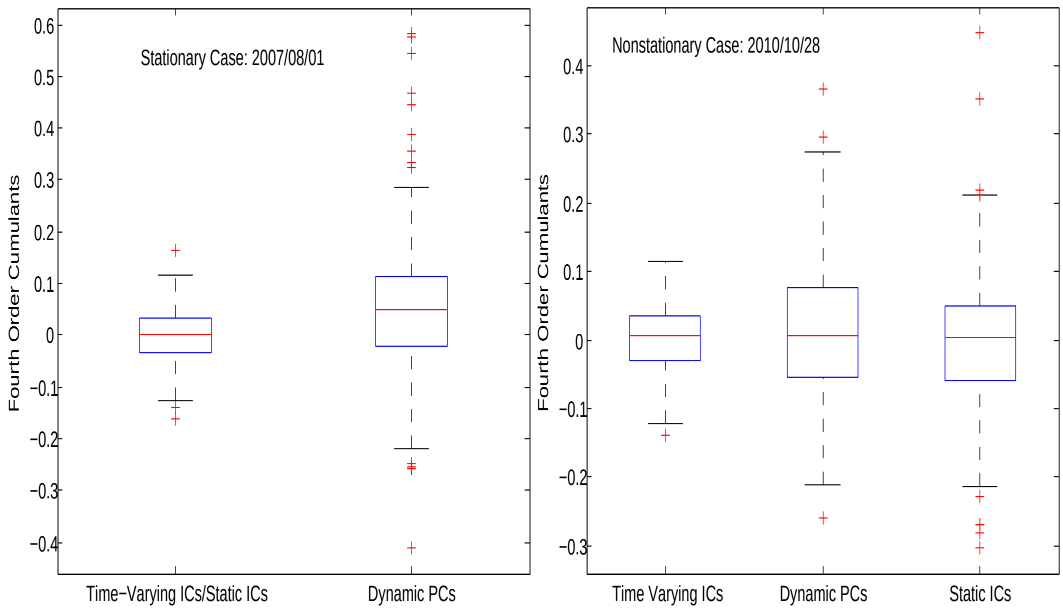

Figure 5: Boxplots of the fourth order cross-cumulants of the time varying ICs, the dynamic PCs and the static ICs (applicable for nonstationary case 2010/10/28).

5 Conclusion and discussion

We proposed the TVICA method to extract independent sources of high dimensional and complex financial time series that are neither Gaussian distributed nor stationary. We allow the ICA linear filter to change over time, and estimate it in more precisely intervals of homogeneity. These "trust" intervals with an approximately constant linear filter are identified via a local change point detection approach. This technique doesn't require the specification of the type, magnitude or stochastic models of change.

Several hyperparameters need to be adjusted. In our simulation study based on real data scenarios, however, we demonstrate that the procedure is robust w.r.t. the choice of the parameters. The TVICA method is easy to implement both under homogeneity and in a situation with different kinds of changes. The obtained ratio of rejecting the null of homogeneity is reasonable and the location of change(s) has been nicely detected. We also conducted the real data analysis. The proposed method detected the intervals of homogeneity that consist of the facts in real life. It is interesting that the method indicates an access of a new homogenous state of financial markets after the recent global financial crisis, though with a possibly different perspective on structure.

Although our study pays attention to just extracting independent sources, extensions of the TVICA method to explicitly account for other important data characteristics and other applications such as risk management and forecasting, are straightforward. Due to independence of the linear filtered sources, univariate models can be adopted for each series of the independent sources.

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